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SEÇÃO – ARTIGOS ORIGINAIS

Uma conexão entre a teoria do consumidor e a teoria quantitativa da moeda: uma abordagem de Cantillon

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Resumo: Este ensaio mostra uma conexão entre a teoria do consumidor e a teoria quantitativa da moeda inspirada na abordagem de Richard Cantillon, segundo a qual mudanças no estoque de moeda influenciam os níveis de preços, caracterizando a elasticidade da oferta de moeda em relação ao nível de preços ao consumidor. Assim, mudanças na oferta de moeda geram mudanças no nível de preços, e tais efeitos monetários afetam a escolha ótima do consumidor se a mudança na oferta de moeda afetar os preços de dois bens com intensidades diferentes. Ao expandir a quantidade de dinheiro na economia, o governo afeta os preços relativos e, consequentemente, altera as decisões dos agentes econômicos em uma economia de mercado. Baseado em dados trimestrais americanos, 1946:04 a 2019:04, os resultados empíricos mostram que a variação do estoque de dinheiro afeta diretamente a variação ótima do consumo e indiretamente afeta a variação ótima do consumo via variação dos preços relativos, evidenciando que o dinheiro não é neutro.

Palavras-chave: teoria do consumidor, efeito Cantillon, TQM, elasticidade da oferta de moeda ao nível de preços ao consumidor, não neutralidade da moeda.

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Connecting the consumer theory and the quantity theory of money: a Cantillon's approach

Abstract: This essay shows a connection between consumer theory and the quantity theory of money inspired by Richard Cantillon's approach, in which changes in the money stock influence price levels, characterizing the elasticity of the money supply to the consumer price level. Thus, changes in the money supply generate changes in the price level. Such monetary effects impact the consumer's optimal choice if the change in the money supply influences the prices of two goods with different intensities. When the government expands the amount of money in the economy, it affects relative prices and, consequently, changes the decisions of economic agents in a market economy. Based on U.S. quarterly data, 1946:04 to 2019:04, the empirical results show that the change in the money stock directly affects the optimal change in consumption and, indirectly, the optimal change in consumption via the change in relative prices, showing that money is not neutral.

Keywords: consumer theory, Cantillon effect, QTM, elasticity of money supply at consumer price level, money non-neutrality.

Conectando la teoría del consumidor y la teoría cuantitativa del dinero: un enfoque de Cantillon

Resumen: Este ensayo muestra una conexión entre la teoría del consumidor y la teoría cuantitativa del dinero, inspirado en el enfoque de Richard Cantillon, donde los cambios en el stock de dinero influyen en los niveles de precios, caracterizando la elasticidad de la oferta monetaria con relación al nivel de precios al consumidor. Así, cambios en la oferta monetaria generan cambios en el nivel de precios, y tales efectos monetarios afectan la elección óptima del consumidor si el cambio en la oferta monetaria afecta los precios de dos bienes con diferente intensidad. Cuando, en una economía de mercado, el gobierno expande la cantidad de dinero afecta los precios relativos y, en consecuencia, cambia las decisiones de los agentes económicos. Con base en datos trimestrales estadounidenses, 1946:04 a 2019:04, los resultados empíricos muestran que el cambio en el stock de dinero afecta directamente el cambio óptimo en el consumo, e indirectamente, afecta el cambio óptimo en el consumo a través del cambio en los precios relativos, mostrando que el dinero no es neutral.

Palabras clave: teoría del consumidor, efecto Cantillon, TQM, elasticidad de la oferta monetaria a nivel de precios al consumidor, no neutralidad monetaria.

Introduction

According to Rothbard (1995), much earlier than Adam Smith (1776), Jean Bodin (2012) was the first to formally attempt to explain the relationship between prices and the money stock. Later, Petty (1988) and John Law (IORIO, 2017) realized that money stimulated economic activity. In turn, John Locke (IORIO, 2017) developed the foundations of the quantitative theory of money (QTM), later known as the Cambridge equation. Locke states that there is a proportional relationship between the quantity of money and the price level, considering the value of transactions and the velocity of money as unchanged. Finally, David Hume (RO-

THBARD, 1995) began to consolidate the quantity theory of money over time, as we know it, resulting in the Irving Fisher and Cambridge approaches, among others.

However, not all pre-classical authors followed Hume's approach, since the relationship between stock money, prices, and output is measured in aggregates. For instance, the indicator for monetary aggregates can be M1 or M2; the general price index may be the GDP implicit deflator or the proxy of aggregate production level (GDP). In this context, this approach is not considering individual consumer and producer decisions, but just a mechanical evaluation of aggregated indicators.

The sophistication of pre-classical authors begins with Richard Cantillon (CANTILLON, 2010), whom Spengler (1954a) considered the first modern economist because he discussed the fundamental question in monetary economics. Whether the abundance of money makes goods and services more expensive, the great challenge is to know how and in what proportion the growth of money stock raises the price level.

According to Bordo (1983), Cantillon stresses the interrelationship between the several components of the equation of exchange (QTM) and the dynamic adjustment of relative prices, besides outputs at a disaggregated level, due to changes in the quantity of money.

Unlike Hume, Cantillon (1775) shows that an increase in the money supply would not affect all industries simultaneously and to the same degree. This change in the money stock would affect various industries over time as a chain reaction. Consequently, this process would change the structure of the price level as well as the relative prices in different intensities.

Cantillon rejects the naive assumption of the quantity theory of money (QTM) that an expansion in the money supply would affect all prices simultaneously and with the same intensity. The Cantillon effect is based on the idea that an increase in the money stock generates changes in price levels, and such changes depend on how new money is injected into the economy. Hence, depending on which sector of the economy the money is injected into first and how it is spent, there is an increase in the quantity of money, and prices will increase depending on how the new money holders decide to spend. It means that changes in the quantity of money change relative prices and interest rates, therefore having no real effects on the economy. This approach is known as the Cantillon effect (CANTILLON, 1775).

For a long time, Cantillon's sophisticated monetary approach was forgotten. Mises, Hayek, and other Austrian economists rescued it in the 1930s (BLAUG, 1995). Since then, his approach has been revisited by several authors such as Hone (1944), Spengler (1954a and 1954b), Bordo (1983), Hébert (1981), Murphy (1985), Thornton (2009a and 2009b), Berg (2012a 2012b), Brown and Thornton (2013), Sieron (2017), among others.

QTM can be summed up with the proportionality theorem, which postulates a direct relationship between the quantity of money and prices, without any lasting changes in the real side of the economy. It was assumed that a change in the amount of money could cause real variations in the short term; however, they would disappear, and only the monetary effects on prices would remain. In the long run, this theorem states there would be a proportional relationship between the increase in the quantity of money and the increase in prices. So, one of the characteristics of this approach is to focus on the general price level, and not to pay attention to changes in relative prices, which is one of the focuses of this article.

One of QTM's assumptions is the dichotomy between relative prices and absolute prices, which is valid if changes in relative prices result from changes in real variables; meanwhile, absolute price variations result from changes in the money supply. This assumption means that given the money supply, the money velocity, and the level of economic activity, changes induced by a real shock in relative prices generate compensatory changes in other relative prices so that the absolute price level remains unchanged. However, nothing guarantees that such changes are offset so that the general price level remains unchanged (MOREIRA et al., 2016).

Moreira et al. (2016) discuss the non-neutrality of money and point out that most theories that try to model money neutrality are explicitly or implicitly connected with the QTM. From this perspective, a given change in the money supply affects real variables during a transition period, that is, until the price level adjusts entirely to a new steady-state equilibrium. Nevertheless, this kind of analysis generally does not consider the microeconomic aspects. It considers the monetary issues a secluded bay, assuming that the marginal utility, value, and prices are not connected. The QTM can confirm that it is based on economic aggregates such as general price level, the velocity of money, and domestic output.

Chena and Huang (2012) evaluate the transmission effects of foreign exchange reserves on price levels from China. In this context, one can substitute foreign exchange reserves for money stock to obtain the elasticity of money supply to the consumer price index. This paper uses this monetary elasticity to know if a change in monetary stock impacts consumer price levels differently. In this case, the elasticity of money supply to the consumer price level is obtained from the QTM. The equations (10) and (11) show the definition of the elasticity of money supply to the consumer price levels, where the data set comes from https://fred.stlouisfed.org/ .

Considering the basic model of consumer theory and the quantity theory of money (QTM), this paper uses Cantillon's approach to connect these theories to show that money is not always a veil, entirely neutral, as the consumer theory assures. In this context, if the elasticity of money supply to the consumer price level is unequal, then money is not neutral. So, this paper aims to develop a very simple model to show that money, under certain conditions, can be non-neutral.

This new consumer theory approach is modeled in the most basic form possible in the next section, based on three steps. The first shows the traditional consumer problem; the second connects this problem to QTM; and, inspired by Cantillon's monetary approach, the third shows that changes in the money supply stock can affect the price level with different intensities, based on the elasticity of money supply regarding the consumer price level.

The contribution of this paper is developing a theoretical model to test Cantillon's propositions based on a quarterly dataset from 1946:04 to 2019:04 for the U.S. economy considering time series empirical models.

Although Richard Cantillon is hailed as the first modern economist to address monetary issues and is even considered by some scholars as the father of modern economics, instead of Adam Smith, there is no theoretical model in the literature to support his ideas regarding the role of currency in the economy. Naturally, there is no intention to incorporate all of Richard Cantillon's complex thinking, but only to use some of his essential contributions to show how the money stock variation has a relevant role in consumer theory.

After this short introduction, a theoretical model is developed in Section 2. Section 3 shows the methodological aspects, while section 4 shows the empirical results. Section 5 discusses the results, and, at last, the conclusions are presented.

The Model

From the basic model of consumer theory and QTM, we elaborated a new model inspired by the work of Richard Cantillon, which incorporates the effect of money on relative prices. Our model contributes to the literature, since, until now, there was no model with microeconomic foundations based on his approach.

Let us assume the usual hypotheses from consumer theory where

$$U = f(X_1, X_2) \quad (1)$$

$$\text{subject to } Y = P_1 X_1 + P_2 X_2 \quad (2)$$

where Y is the nominal income. Considering that Y = P.Q, where P is the general price index and Q is the real output or real income, it is possible to obtain a connection between the budget constraint and the QTM.

The QTM states that

$$MV = PQ \quad (3)$$

where M is the money stock, and V is the velocity of money circulation assumed constant. Taking equations (2) = (3), we have

$$MV = PQ = Y = P_1 X_1 + P_2 X_2 \quad (4)$$

so that the budget constraint is redefined as

$$MV = P_1 X_1 + P_2 X_2 \quad (5)$$

Solving the consumer problem from equation (1) subject to equation (5), we still obtain the same expression that is known as the marginal rate of substitution (MRS) between goods 1 and 2, as follows

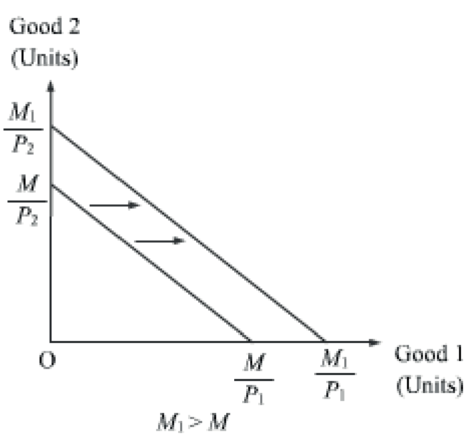
$$MRS_{X1,\,X2} = -\,(dX_1 / dX_2) = Umg(_2) / Umg(_1) = P_2 / P_1 \quad (6)$$

where Umg is the marginal utility.

In this context, this result does not change because the consumer theory assumes the same statements from QTM, i.e., that a change in the money stock affects all prices in the economy simultaneously and in the same proportion. This assumption from QTM is associated with the dichotomy between absolute prices and relative prices. In other words, relative prices are explained only by changes in real variables, while absolute prices are explained only by changes in monetary variables.

Let us assume an exogenous increase in consumer income or a proportional increase in the money stock, given the velocity of money circulation. In this case, the budget line moves up to the right, and its slope remains unchanged, as well as the relative prices, because they increase in the same proportion. Thus, it does not matter whether there is an exogenous increase in the representative consumer's income or an increase in the same proportion on the money stock. Both cases imply that money is still neutral.

Figure 1 - The budget line



From now on, let us consider the Cantillon effect. The relevant issue is showing how changes in the money stocks affect the traditional consumer optimization problem, considering Cantillon's approach. Hence, one introduces the budget constraint based on equation 5, so that

$$P_2 = (MV - P_1 X_1) / X_2 \quad (7)$$

In this sense, we can show the effect of money on P_2, given P_1, as follows

$$dP_2 / dM = V / X_2 \quad (8)$$

and considering that $V = PQ / M = Y / M$, we obtain

$$dP_2 / dM = (1/X_2)*(Y / M) \qquad (9)$$

Based on Equation (9), we can obtain the elasticity of money supply to the consumer price levels ε_2 and ε_1 as follows

$$\varepsilon_2 = [dP_2 / dM]*(M/P_2) = [(1/X_2)*(Y / M)]*(M/P_2) \qquad (10)$$

By analogy, we also have that

$$\varepsilon_1 = [dP_1 / dM]*(M/P_1) = [(1/X_1)*(Y / M)]*(M/P_1) \qquad (11)$$

As already mentioned, the Cantillon effect ensures that changes in prices resulting from changes in the money supply depend on how the newly created money enters the economy. The new money gradually spreads in the economic system, consequently changing relative prices (SIERON, 2017).

Taking this into account, based on Cantillon's approach, we expect that $\varepsilon_1 \neq \varepsilon_2$, since the change in monetary stock does not have the same impact on prices P_1 and P_2. As both prices change in unequal proportions, the relative price P1/P2 changes. In other words, if $\varepsilon_1 = \varepsilon_2$, money is neutral, as predicted by the standard consumer theory. However, if $\varepsilon_1 \neq \varepsilon_2$, money is not neutral, as predicted by Cantillon's approach.

Based on the equations (10) and (11) and considering that $Y = Y_2 = Y1$, then we have that

$$Y2 = P_2*X_2*\varepsilon_2 \qquad (12)$$

and by analogy, we also have that

$$Y_1 = P_1*X_1*\varepsilon_1 \qquad (13)$$

Given that $MV = PQ = Y$ and considering the equations (12) and (13), we can show that

$$MV = Y_2 = P_2*X_2*\varepsilon_2 \qquad (14)$$

$$\text{and } M = (1/V)*P_2*X_2*\varepsilon_2 \qquad (15).$$

By analogy we have

$$MV = Y_1 = P_1*X_1*\varepsilon_1 \qquad (16)$$

$$\text{and } M = (1/V)*P_1*X_1*\varepsilon_1 \qquad (17)$$

where subscripts 1 and 2 from Y_1 and Y_2 refer to QTM where $M = (1/V)*Y$, such that $M = (1/V)*Y_1$ is associated with ε_1, while $M = *(1/V)Y_2$ is associated with ε_2 respectively.

Given our considerations, let us show the consumer optimization problem presented in equations (1), (5), (15) and (17) as follows:

$U = f(X_1, X_2)$

subject to $M = (1/V)*P_{1*}X_1 + (1/V)*P_2*X_2$

$M = (1/V)*P_1*X_1*\varepsilon_1$

$M = (1/V)*P_2*X_2*\varepsilon_2$

In order to solve the first order condition, we have

$$V = U(X_1, X_2) + \gamma_1 [M - (1/V)*P_{1*}X_1 - (1/V)*P_2*X_2] + \gamma_2 [M - (1/V)*P_1*X_1*\varepsilon_1] + \gamma_3 [M - (1/V)*P_2*X_2*\varepsilon_2] \quad (18)$$

where one assumes that $\gamma_1 = \gamma_2 = \gamma_{3\,=}\,\gamma$.

Thus, the optimization process occurs when calculating the derivative of the utility function, $U = f(X_1, X_2)$, subject to three constraints: i) $M = (1/V)*P_1*X_1 + (1/V)*P_2*X_2$, ii) $M = (1/V)*P_1*X_1*\varepsilon 1$ and iii) $M = (1/V)*P_2*X_2*\varepsilon_2$. Considering that $\gamma_1 = \gamma_2 = \gamma 3 = \gamma$, equation (18) is derived regarding to $_{X1}$ and X_2.

Finally, based on the first-order conditions, we obtain the following results:

$$MRS_{X2,\,X1} = -(dX_2 / dX_1) = U'(X_1) / U'(X_2) = P_1 (1 + \varepsilon_1) / P_2 (1 + \varepsilon_2) \quad (19)$$

Hence, if $\varepsilon_1 = \varepsilon_2$, then $U'(x_1) / U'(x_2) = P_1 / P_2$. This is the case predicted by the standard consumer theory, in which money is neutral. However, if the elasticities are different, i.e., $\varepsilon_1 \neq \varepsilon_{2,}$ money is not neutral. Thus, changes in the money stock affect the budget line inclination and relative prices, as predicted by Cantillon's approach. In this case, money is not neutral.

Methodological aspects

The first part shows Table 1 description of the variables from the FRED database, while the second one shows the transformed variables used in the empirical models.

Table 1 – Database (Data from 1946:04 to 2019:04)

Variables code from FRED	Variable's description
PCECC96	Real Personal Consumption Expenditures, Billions of Chained 2012 Dollars, Quarterly
GDPC1	Real Gross Domestic Product, Billions of Chained 2012 Dollars, Quarterly
CURRCIR	Currency in Circulation, Billions of Dollars, Monthly
DPIC96	Real Disposable Personal Income, Billions of Chained 2012 Dollars, Quarterly
B230RC0Q173SBEA	Population, Thousands, Quarterly
CPIAUCNS	Consumer Price Index for All Urban Consumers: All Items in the U.S. City Average, Index 1982-1984=100, Monthly
CUUR0000SAF113	Consumer Price Index for All Urban Consumers: Fruits and VegeTables in the U.S. City Average, Index 1982-1984=100, Monthly
CWUR0000SAF1	Consumer Price Index for All Urban Wage Earners and Clerical Workers: Food in the U.S. City Average, Index 1982-1984=100, Monthly
CPIAPPSL	Consumer Price Index for All Urban Consumers: Apparel in the U.S. City Average, Index 1982-1984=100, Monthly
CWUR0000SEHF	Consumer Price Index for All Urban Wage Earners and Clerical Workers: Energy Services in the U.S. City Average, Index 1982-1984=100, Monthly
Variables	**Transformed variables description**
l_Real_gdp_1	log (GDPC1 / GDPC1 (-1))
l_Real_pers_cons_exp_1	log (PCECC96 / PCECC96 (-1))
real_dis_pers_income_1_pc	(DPIC96) / (B230RC0Q173SBEA)
l_real_dis_pers_income_1_pc	Log (real_dis_pers_income_1_pc) / real_dis_pers_income_1_pc(-1))
Money_pc	(CURRCIR) / (B230RC0Q173SBEA)
l_money_pc	log (Money_pc / Money_pc(-1))
RP_Apparel_CPI _	(CPIAPPSL / CPIAUCNS)
l_RP_Apparel__CPI	log (RP_Apparel_CPI / RP_Apparel_CPI(-1))
RP_Fruits_Veg_CPI	(CUUR0000SAF113 / CPIAUCNS)
l_RP_ Fruits_Veg_CPI	log (RP_Fruits_Veg_CPI / RP_Fruits_Veg_CPI(-1))
RP_Food_CPI _	(CWUR0000SAF1 / CPIAUCNS)
l_RP_Food__CPI	log (RP_Food_CPI / RP_Food_CPI(-1))
RP_Energy_CPI _	(CWUR0000SEHF / CPIAUCNS)
l_RP_Energy__CPI	log (RP_Energy_CPI / RP_Energy_CPI(-1))

Data source - Federal Reserve Bank of St. Louis. Note (1): All the billion values are transformed into thousands. Note (2): Monthly variables were transformed into quarterly ones. Note (3): RL = Relative price and l = log.

The descriptive statistics are shown in Table 2, in which each variable is identified in capital letters from A to H to facilitate the presentation of the correlation matrix in Table 3.

Table 2 – Descriptive statistics

Variables	Mean	Median	Maximum	Minimum	Std. Dev.
l_Real_gdp_1 **(A)**	0.007726	0.007462	0.038549	-0.026301	0.009288
l_Real_pers_cons_exp_1 **(B)**	0.008072	0.007949	0.050077	-0.030610	0.007987
l_money_pc **(C)**	5.947003	5.290292	30.81094	-20.84834	5.742031
l_RP_Apparel__CPI **(D)**	-0.00444	-0.003429	0.088567	-0.090865	0.037829
l_RP_ Fruits_Veg_CPI **(E)**	0.000607	0.003537	0.130907	-0.175695	0.049058
l_RP_Food__CPI **(F)**	-0.00019	-0.000406	0.036616	-0.026367	0.010396
l_RP_Energy__CPI **(G)**	-0.00028	-0.002508	0.108460	-0.091711	0.029245
d_optimal_consumption **(H)**	0.08390	-0.124232	13.07395	-11.26882	2.840614

Note: 291 observations.

The correlation matrix displays that there are no strong correlations between the variables.

Table 3 – Correlation Matrix

	A	B	C	D	E	E	G	H
A	1.000	0.582	-0.046	0.038	0.033	0.059	-0.035	0.003
B		1.000	0.005	-0.035	0.086	0.037	0.072	-0.032
C			1.000	-0.379	0.179	0.245	0.107	0.206
D				1.000	-0.206	-0.243	-0.659	-0.103
E					1.000	0.542	-0.068	0.179
F						1.000	-0.093	0.200
G							1.000	-0.257
H								1.000

Note: l_Real_gdp_1 = (A), l_Real_pers_cons_exp_1= (B), l_money_pc = (C), l_RP_Apparel__CPI = (D), l_RP_ Fruits_Veg_CPI = (E), l_RP_Food__CPI = (F), l_RP_Energy__CPI = (G), d_optimal_consumption = (H)

The empirical approach is implemented in four steps. The **first step** shows a non-linear relationship between real GDP and real spending on personal consumption. In other words, the goal is to determine the value of the variable l_real_pers_cons_exp_1 that maximizes l_real_gdp_1 based on equation 20, considering estimates with the Generalized Method of Moments (GMM).

$$(\text{l\_real\_gdp\_1})_t = \lambda o + \lambda_1{}^*(\text{ l\_real\_gdp\_1})_{t-1} + \lambda_2{}^*(\text{ l\_real\_pers\_cons\_exp\_1})_t + \lambda_3{}^*(\text{l\_real\_pers\_cons\_exp\_1})\ 2_t + u_t \qquad (20)$$

where λ_i is the regression's parameters and u_t is the error term. It is expected that the estimated parameters λ_1 and λ_2 are, respectively, positive and negative, i.e., $\lambda_1 > 0$ and $\lambda_2 < 0$. In this case, a concave curve can be obtained in an inverted U shape. Thus, the optimal value of real GDP that maximizes real spending on personal consumption can be determined. In this context, since equation 20 is estimated, one can derive this equation based on the predicted values as follows:

$$\text{Predicted\_L\_Real\_GDP} = \lambda o + \lambda_1 * (\text{l\_real\_gdp\_1})_{t-1} + \lambda_2 * (\text{l\_real\_pers\_cons\_exp\_1})_t + \lambda_3 * (\text{l\_real\_pers\_cons\_exp\_1})\, 2_t \text{ where } \lambda_i \text{ are the estimated coefficients.} \quad (21)$$

Thus, based on equation 21, optimal consumption can be obtained by deriving the predicted value in relation to real spending on personal consumption. In this case, we have the first-order condition

$$= \lambda_2 + 2 * \lambda_3 * (\text{l\_real\_pers\_cons\_exp\_1}) = 0 \quad (22)$$

and, therefore, it is possible to find the value of the variable "optimal consumption". This optimal value represents the optimal consumption from 1946:04 to 2019:04.

The second step is to generate the dependent variable. In this case, the optimal consumption value is divided by the population of each year to obtain an optimal consumption value per capita year by year. Finally, we obtain the variation of this variable to make it stationary, calling it d_optimal_consumption.

The third step shows the effect of money *per capita* change on the optimal consumption variation, d_optimal_consumption based on OLS Robust Method, as follows:

$$(\text{d\_optimal\_consumption})_t = \beta_{0+}\beta_1(\text{d\_optimal\_consumption})_{t-1} + \beta_2(\text{d\_optimal\_consumption})_{t-2} + \dots .+ \beta_n(\text{d\_optimal\_consumption})_{t-n} + \varrho_0(\text{l\_money\_pc})_t + \varrho_1(\text{l\_money\_pc})_{t-1\,+}\, \varepsilon_t \quad (23)$$

In this case, it is possible to evaluate the effect of per capita money variation on the optimal consumption change. If the estimated coefficients, ϱ_0 or ϱ_1, are statistically significant, it implies that money will not be neutral.

At last, **the fourth step** displays the Cantillon effect based on simultaneous equation models via GMM, such as

$$(\text{Relative prices})_t = \alpha o + \alpha_1(\text{Relative prices})_{t-1} + \alpha_2(\text{l\_money\_pc})\, t + \alpha_3(\text{l\_money\_pc})_{t-1} + Z_t \quad (24)$$

and

$$(\text{d\_optimal\_consumption})_t = \beta_{0+}\beta_1(\text{d\_optimal\_consumption})_{t-1} + \beta_2(\text{d\_optimal\_consumption})_{t-2} + \dots .+ \beta_n(\text{d\_optimal\_consumption})_{t-n} + \delta_0(\text{l\_money\_pc})_t + \delta_1(\text{l\_money\_pc})_{t-1} + \delta_1(\text{Relative prices})_t + \delta_2(\text{Relative prices})_{t-1+}\, \varepsilon_t \quad (25)$$

It is possible to calculate the indirect effect from "l_money_pc" variable based on equation 24 on the dependent variable, d_optimal_consumption, based on equation 25, via relative prices. In this case, the indirect effect is given by $(\alpha 2)*(\delta 1)$, for the current time, or $(\alpha 3)*(\delta 2)$, for lagged time. It means that variations in the per capita money stock affect relative prices in equation 24, which affects the variations in optimal consumption as observed in equation 25.

In addition to the methods of empirical models, such as RLS and GMM, other methods are used in the robustness tests section, such as Quantile Regression, Stepwise Regression, and ARDL Methods. The robust least squares (RLS) method refers to a variety of regression methods, which are *robust*, or less sensitive to outliers. There are some RLS methods to determine a regression model: M-estimation (HUBER, 1973), S-estimation (ROUSSEEUW; YOHAI, 1984), and MM-estimation (YOHAI, 1987).

Koenker and Bassett (1978) originally said that quantile regression provides estimates of the linear relationship between regressors X and a specified quantile of the dependent variable Y. Quantile regression generates a more relevant description of the conditional distribution than conditional mean analysis alone, allowing us, for example, to describe how the regressor variables affect the median, or perhaps the 10th or 95th percentile of the response variable. Moreover, since the quantile regression approach does not require strong distributional assumptions, it offers a robust method for modeling these relationships.

There is extensive literature describing the benefits and pitfalls of stepwise regression. Without making any recommendations ourselves, we refer the reader to Derksen and Keselman (1992), Roecker (1991), and Hurvich and Tsai (1990).

Autoregressive Distributed Lag (ARDL) models are standard least squares regressions that include lags of both the dependent variable and explanatory variables as regressors (GREENE, 2008). Although ARDL models have been used in econometrics for decades, they have gained popularity in recent years as a method of examining cointegrating relationships between variables through the works of Pesaran and Shin (1998) and Pesaran, Shin, and Smith (2001).

Moreira et al. (2016) also estimate a system of two regressions via GMM to avoid endogeneity problems using instrumental variables (IV). The IV must be good in order to be relevant and valid. The authors use the test of over-identifying Sargan-Hansen, also known as the J-statistic. Besides, the Stock-Yogo test evaluates the null hypothesis, in which the instruments are weak, based on the Cragg-Donald test. In this context, see Moreira (2001), Stock, Wright and Yogo (2002), or Stock and Yogo (2004). Moreover, they use the procedure of Newey and West (1987a; 1987b) for all estimated models to minimize problems of unknown heteroskedasticity and serial correlation of residuals. Hence, to test this article's empirical results` robustness, we follow Moreira et al. (2016), who show the methodological aspects in more detail.

Empirical results

Table 4 exhibits times series unit root tests and reveals that all the variables are stationaries.

Table 4 - Unit root tests (H0: time series has unit root)

	Augmented Dickey-Fuller test statistic (ADF)			**Phillips-Perron test statistic (P.P.)**		
Variables	**Critical value: 5% level**	**t-Statistic**	**p-value**	**Critical value: 5% level**	**Adj. t-Stat**	**p-value**
l_Real_gdp_1	-2.8712	-11.677	<0.0001	-2.8712	-11.526	<0.0001
l_Real_pers_cons_exp_1	-2.8713	-8.2145	<0.0001	-2.8712	-16.147	<0.0001
l_Money_pc	-2.8714	-3.4631	0.0097	-2.8712	-20.908	<0.0001
l_RP_Apparel__CPI	-2.8717	-2.8905	0.0477	-2.8712	-51.635	0.0001
l_RP_ Fruits_Veg_CPI	-2.8718	-5.0772	<0.0001	-2.8712	-37.101	0.0001
l_RP_Food__CPI	-2.8718	-3.8966	0.0024	-2.8712	-17.876	<0.0001
l_RP_Energy__CPI	-2.8718	-3.2495	0.0183	-2.8712	-23.628	<0.0001
d_optimal_consumption	-2.8718	-5.3335	<0.0001	-2.8713	-24.740	<0.0001

Note: Include Constant.

Table 5 shows the empirical results with three different models based on Robust Least Squares Methods according to equation 20. The estimated constant terms are not statistically significant for all models, and models 1 and 3 show better results due to higher Adjusted R--squared than the result from model 2. The models 1 and 3 present the estimated coefficients from “l_real_pers_cons_exp_1” and “(l_real_pers_cons_exp_1)^2” with signs positive and negative respectively. These results show a non-linear relation between the variables “l_real_gdp_1” and “l_real_pers_cons_exp_1”, which implies a concave curve, i.e., a curve in U inverted shape, according to models 1 and 3. However, considering just models 1 and 3, while the estimated coefficients from variable “l_real_pers_cons_exp_1” are statistically significant at 1% level, the estimated coefficients from variable (l_real_pers_cons_exp_1)^2 is marginally significant at 10% level.

Table 5 – Empirical Results: OLS Robust Method (1946, p. 04; 2019, p. 04)

Dependent variable: l_real_gdp_1	**MODEL 1- Robust Least Squares: (M-estimation)**		**MODEL 2 – Robust Least Squares: (S-estimation)**		**MODEL 3 - Robust Least Squares: (MM-estimation)**	
Variable	Coefficient (Std.Error)	P-value	Coefficient (Std.Error)	P-value	Coefficient (Std.Error)	P-value
Constant	-7.33E-06 (0.000572)	0.9898	0.000879 (0.000709)	0.2147	-6.05E-06 (0.000572)	0.9916
l_real_gdp_1 (-1)	0.141820 (0.041999)	< 0.00001	-0.006328 (0.052000)	0.9031	0.141664 (0.041996)	0.0007 .
l_real_pers_cons_ exp_1	0.867926 (0.054757)	< 0.00001	0.927710 (0.067795)	< 0.00001	0.868160 (0.054752)	<0.00001 .
(l_real_pers_cons_ exp_1)2	-3.802327 (2.120728)	0.0730	-3.084353 (2.625693)	0.2401	-3.805116 (2.120563)	0.0728 .
Other Statistics						
Adjusted R-squared	0.318381		0.195377		0.317780	
Adjust Rw-squared	0.573551				0.574245	

Note: Model 2 does not present the Adjusted R-squared statistic.

To avoid possible endogeneity problems, it is convenient to use a GMM approach, and still, based on equation 20, Table 6 shows that all estimated coefficients are statistically significant at a 5% level. Besides, the estimated coefficients from "l_real_pers_cons_exp_1" and "$(\text{l\_real\_pers\_cons\_exp\_1})^2$" present signs positive and negative respectively as well. This result also shows a non-linear relation between the variables "l_real_gdp_1" and "l_real_pers_cons_exp_1", which implies a concave curve, i.e., a curve in U inverted shape.

Table 6 – Empirical Results: (1946, p. 04; 2019, p. 04)

Dependent variable: l_real_gdp_1	Method: Generalized Method of Moments: GMM			
Variable	Coefficient	Std. Error	t statistic	p-value
Constant	-0.001950	0.000464	-4.199994	0.0000
l_real_gdp_1 (-1)	0.240549	0.046157	5.211564	0.0000
l_real_pers_cons_exp_1	1.092159	0.116141	9.403756	0.0000
$(\text{l\_real\_pers\_cons\_exp\_1})^2$	-11.19564	5.017772	-2.231197	0.0265
Statistics				
Adjusted R-squared = 0.461584			J – Statistics (p-value) = 0.982464	
Instruments: l_real_gdp_1(-2to-17), l_real_pers_cons_exp_1(-1to-16), l_real_dis_pers_income_1, l_real_dis_pers_income_1(-1to-15).				

We can observe that the Adjusted R-squared from Table 6 is higher than the models from Table 5, and, additionally, the empirical results from Table 6 use IV via GMM. Hence, such aspects justify choosing the model from Table 6 as the more appropriate. In this sense, taking into account the predicted value of the dependent variable "l_real_gdp_1", it is possible to calculate the optimal consumption level based on equation 21 as follows:

$$\textit{Predicted\_L\_Real\_GDP\_1} = -0.001950 + 0.240549*(\textit{l\_real\_gdp\_1})_{t-1} + 1.092159*(\textit{l\_real\_pers\_cons\_exp\_1})_t - 11.19564*(\textit{l\_real\_pers\_cons\_exp\_1})^2_t$$

In this case, the first-order condition, based on equation 22, is shown as follows:

$$(d(\textit{Predicted\_L\_Real\_GDP}))/(d(\textit{l\_real\_pers\_cons\_exp\_1})) = 1.092159 + 2*(-11.19564)*(\textit{l\_real\_pers\_cons\_exp\_1}) = 0$$

In which it is possible to find out the value of the variable "optimal consumption" such that "*L _REAL_PERS_CONS_EXP_1*" = 0.0487.

Based on the second step, the optimal consumption value (0.0487) is divided by the population of each year to obtain an optimal consumption value per capita year by year. After that, we obtain the variation of this variable, "*d_optimal_consumption*", to make it stationary, according to Table 4.

The third step shows the impact of money per capita change on the optimal consumption variation, "d_optimal_consumption," according to equation 23. Furthermore, the regressions are based on Robust Least Squares models, as shown in Table 7.

The empirical results show that all estimated coefficients, except the constant term, are statistically significant at a 10% level. The variable of interest, l_money_pc, based on models 1 and 3, reveals that the estimated coefficients are statistically significant at a 1% level, and that this variable presents a directly proportional effect on the optimal consumption variation. Model 2 also shows a directly proportional effect on the optimal consumption variation, although the estimated coefficient is marginally significant at a 10% level.

Table 7 – Empirical Results: OLS Robust Method (1946:04 to 2019:04)

Dependent variable: d_optimal_consumption	**MODEL 1- Robust Least Squares: (M-estimation)**		**MODEL 2 – Robust Least Squares: (S-estimation)**		**MODEL 3 - Robust Least Squares: (MM-estimation)**	
	Coefficient (Std.Error)	P-value	Coefficient (Std.Error)	P-value	Coefficient (Std.Error)	P-value
Constant	-0.089073 (0.081916)	0.2769	0.103947 (0.092058)	0.2588	-0.088608 (0.081801)	0.2787
d_optimal_consumption(-1)	-0.173055 (0.037085)	< 0.00001	-0.172766 (0.041676)	<0.00001	-0.173339 (0.037033)	< 0.00001 .
d_optimal_consumption(-2)	-0.525258 (0.038795)	< 0.00001	-0.511338 (0.043597)	< 0.00001	-0.525691 (0.038740)	<0.00001 .
d_optimal_consumption(-3)	-0.165353 (0.038468)	< 0.00001	-0.190291 (0.043231)	< 0.00001	-0.165370 (0.038414)	<0.00001 .
d_optimal_consumption(-4)	0.403628 (0.041436)	< 0.00001	0.451394 (0.046566)	< 0.00001	0.403128 (0.041378)	<0.00001 .
l_money_pc	0.027438 (0.010332)	0.0079	0.019923 (0.011612)	0.0862	0.027348 (0.010318)	0.0080 .
			Other Statistics			
Adjusted R-squared	0.643688		0.642305		0.641389	
Adjust Rw-squared	0.901938				0.902858	

Note: Model 2 does not present the Adjusted R-squared statistic.

It does not matter if the impact of money shows a direct or even inversely proportional relation to the optimal consumption, because whatever the sign of the estimated coefficient, being statistically different from zero, it implies that money matters, i.e., that money is not neutral. Moreover, it is reasonable to expect that the effect of the money change can be lagged by optimal consumption. In other words, money stock variations in previous periods may influence the optimal current consumption. Therefore, according to the Cantillon approach, these results display empirical evidence that money affects the optimal level of consumption.

Robustness tests

Considering equation 23, Table 8 exhibits three different methods, i.e., quantile regression, stepwise regression, and ARDL model. The empirical results show that the variable of interest, "l_money_pc," reveals estimated coefficients at a 1% level based on models 2 and 3. This variable shows a directly proportional effect on the optimal consumption variation.

Model 1 also shows a directly proportional effect of l_money_pc on the optimal consumption variation, although the estimated coefficient is marginally significant at the 10% level. The results are similar to Table 7, except for the case of model 3 – ARDL method, which includes a lagged variable, "l_money_pc (-1)". In this case, the estimated coefficient is statistically significant at a 5% level, but with a negative sign.

Table 8 – Empirical Results: (1946:04 to 2019p. 04)

Dependent variable: d_ optimal_consumption	MODEL 1 - Method: Quantile Regression (Median)		MODEL 2 – Method: Stepwise Regression		MODEL 3 - Method: ARDL	
	Coefficient (Std.Error)	P-value	Coefficient (Std.Error)	P-value	Coefficient (Std.Error)	P-value
Constant	-0.008627 (0.107661)	0.9362	-0.137972 (0.108079)	0.2028	0.031981 (0.126047)	0.7999
d_optimal_ consumption(-1)	-0.149607 (0.056573)	0.0086	-0.082238 (0.048930)	0.0939	-0.033602 (0.052066)	0.5192 .
d_optimal_ consumption(-2)	-0.483946 (0.073722)	< 0.00001	-0.442824 (0.051185)	< 0.00001	-0.462627 0.051278)	<0.00001 .
d_optimal_ consumption(-3)	-0.170992 (0.067343)	0.0117	-0.080481 (0.050754)	0.1139	-0.038701 (0.052859)	0.4647 .
d_optimal_ consumption(-4)	0.480731 (0.084778)	< 0.00001	0.538520 (0.054670)	< 0.00001	0.511360 (0.055175)	<0.00001
L_money_pc	0.024347 (0.014172)	0.0869	0.039810 (0.013632)	0.0038	0.045159 (0.013662)	0.0011 .
L_money_pc(-1)					-0.034383 (0.013469)	0.0112 .
			Statistics			
Adjusted R-squared	0.604597		0.826213		0.829571	

All the models from Tables 7 and 8 show that the money stock variation affects the optimal consumption change regardless of the estimated coefficient signs and its lags.

Finally, the fourth step displays the Cantillon effect based on simultaneous equation models via GMM, according to equations 24 and 25. Tables 9 and 10, based on a GMM system of two equations, show four simultaneous equations systems:

i) the first system shows the relation of two equations so that Model 1A (Table 9) is connected with Model 1B (Table 10); the second displays the connection between Model 2A (Table 9) and Model 2B (Table 10); iii) the third, the interaction between Model 3A (Table 9) and Model 3B (Table 10); and, last, the fourth system exhibits the connection between Model 4A (Table 9) and Model 4B (Table 10).

Table 9 shows that all the estimated coefficients are statistically significant at the 1% level, except for the variable "l_money_pc" from Model 4A, which is marginally significant at a 10% level. Considering model 1A, it is worth noting that the interest variable "l_money_pc" presents an impact inversely proportional to the dependent variable "l_apparel__cpi" with an estimated coefficient value of -0.000210.

In addition, the statistic J does not reject the hypothesis that the instruments are good. In turn, the Stock-Yogo test, based on the Cragg-Donald F statistic, does not accept the null hypothesis that the instruments are weak. Hence, the instruments are good and valid. These results are similar to models 2A, 3A, and 4A.

Table 9 also shows that the empirical results validate the Cantillon effect, since the variation in the per capita currency stock impacts relative prices with an elasticity other than 1 or -1, according to the Wald test presented in Table 9. The null hypothesis assumes that the estimated coefficient of currency variation per capita is statistically equal to 1 or -1. As the null hypothesis is not accepted, this means that a monetary shock affects prices with different intensities confirming Cantillon's approach and refuting the QTM assumption.

Table 9 - Estimation Method: GMM (Quarterly data from 1959:04 - 2019:04)

		Dependent variables			
		l_apparel__cpi	l_fruits_veg_cpi	l_food__cpi	l_energy__cpi
		Model 1A	Model 2A	Model 3A	Model 4A
Constant	C(1)	-0.007344*** (0.000519)	-0.008831*** (0.000969)	0.000785*** (0.000161)	0.005916*** (0.001602)
l_money_pc	C(2)	-0.000210*** (4.47E-05)	0.001576*** (0.000107)	0.000412*** (1.82E-05)	-0.000157* (8.14E-05)
l_money_pc(-1)	C(3)			-0.000595*** (2.07E-05)	-0.000898*** (0.000106)
l_apparel_cpi(-1)	C(4)	-0.956954*** (0.004268)			
l_fruits_veg_cpi (-1)	C(5)		-0.261784*** (0.008120)		
l_food__cpi(-1)	C(6)			0.058935*** (0.020185)	
l_energy__cpi(-1)	C(7)				-0.549260*** (0.008625)
		Statistics			
Adjusted R-squared		0.860756	0.106425	0.129147	0.096511
J – Statistics (p-value)		0.118267	0.213065	0.533722	0.154188
Stock-Yogo test (critical value at 5%)		20.76	21.42	20.86	19.86
Cragg-Donald F-stat:		73.46657	61.07775	33.70795	20.00454
Wald test p-value(Chi-Square)		Ho: C(2) = -1 Prob. = <0,00001	Ho: C(2) = 1 Prob. = <0,00001	Ho: C(2) = 1 Prob. = <0,00001	Ho: C(2) = -1 Prob. = <0,00001
Wald test p-value(Chi-Square)				Ho: C(3) = 1 Prob. = <0,00001	Ho: C(3) = -1 Prob. = <0,00001
Instruments Model 1A		d_optimal_consumption(-5to-10), l_real_dis_pers_income_1(-1to-5) , l_money, l_money(-1to-10), l_apparel__cpi (-2to-5), C			
Instruments Model 2A		d_optimal_consumption(-5to-7),l_real_dis_pers_income_1(-1to-3),l_money,l_money(-1to-3),l_food__cpi (-2to-3),l_apparel__cpi, l_apparel__cpi (-1to-17),l_fruits_veg_cpi(-2)			
Instruments Model 3A		d_optimal_consumption(-5to-7),l_real_dis_pers_income_1(-1to-3),l_money,l_money(-1to-3),l_food__cpi (-2to-3),l_apparel__cpi, l_apparel__cpi (-1to-18),C			
Instruments Model 4A		d_optimal_consumption(-5to-7),l_real_dis_pers_income_1(-1to-3),l_money(-1to-10),money, l_money,l_energy__cpi(-2to-3),l_apparel__cpi (-1to-3)			

*Note 1: *** = p-value < 0,01; ** = 0.01 < p-value < 0.05; * = 0.05 < p-value < 0.10. Note 2: Based on Wald tests, if the estimated coefficients of l_money_pc and l_money_pc(-1) are different from 1 or -1, then money affects relative prices. Observe that null hypothesis states that the estimated coefficients of l_money_pc and l_money_pc(-1) are equal to 1 or -1.*

The empirical results from Table 10 show that only the estimated coefficient of the variable "d_optimal_consumption (-3)" from Model 4B is not statistically significant. However, all other estimated coefficients are statistically significant at the 5% level, except the estimated coefficient of the variable “L_money_pc (-1)” from Model 4B, which is marginally significant at the 10% level.

It should be noted that the variable of interest in Table 10, Model 1B, is the relative price “l_apparel_cpi,” which shows an impact inversely proportional to the dependent variable “d_optimal_consumption” with an estimated coefficient value of -3.645064.

Table 10 - Estimation Method: GMM (Quarterly data from 1959:04 - 2019:04)

	Dependent variable: d_optimal_consumption for all models			
	Model 1B	Model 2B	Model 3B	Model 4B
Constant	0.165914*** (0.039979)	0.248397*** (0.034367)	0.273184*** (0.023519)	0.064527** (0.025483)
d_optimal_ consumption (-1)	-0.242729*** (0.051071)	-0.207332*** (0.031011)	-0.270287*** (0.022956)	-0.053424** (0.026521)
d_optimal_ consumption (-2)	-0.467221*** (0.051411)	-0.329811*** (0.039769)	-0.372301*** (0.028362)	-0.203484*** (0.037761)
d_optimal_ consumption (-3)	-0.241958*** (0.053204)	-0.233425*** (0.032965)	-0.286750*** (0.023087)	-0.026771 (0.031894)
d_optimal_ consumption (-4)	0.562838*** (0.052426)	0.706363*** (0.043904)	0.667603*** (0.032307)	0.828934*** (0.043307)
L_money_pc (-1)	-0.008725*** (0.002666)	-0.020377*** (0.002972)	-0.022199*** (0.002216)	-0.005476* (0.003189)
l_apparel_cpi	-3.645064*** (0.807649)			
l_fruits_veg_cpi		-3.503508*** (0.771948)		
l_food__cpi			-25.72996*** (3.727529)	
l_energy__cpi(-1)				-6.698230*** (0.446100)
Indirect effect	(-0.000210)* (-3.645064) = 0.000765	(0.001576)* (-3.503508) = -0.005521	(0.000412)* (-25.72996) = 0.010601	(-0.000898)* (-6.698230) = 0.006624
	Statistics			
Adjusted R-squared	0.816397	0.810730	0.797760	0.803634
No Observations	280	274	273	281

*Note 1: *** = p-value < 0.01; ** = 0.01 < p-value < 0.05; * = 0.05 < p-value < 0.10.*

According to Table 9, based on equations 24 and 25, we can calculate the indirect effect from the current or lagged variable “l_money_pc,” on the dependent variable d_optimal_consumption associated with Table 10, via current or lagged relative prices. In this context, the final indirect effect is calculated by the product of the variable of interest in Table 9 and Table 10; that is, it is calculated by the product between the estimated coefficients of the variables money per capita (Table 9) and relative prices (Table 10).

Hence, "l_money_pc" or "l_money_pc(-1)" variables in Table 9 affects "Relative prices" and, in turn, "Relative prices" influences d_optimal_consumption in Table 10. Notice that the relative prices are presented in both Tables; therefore, they connect models from Tables 9 and 10, revealing the indirect effect.

In this case, based on models 1A (Table 9) and 1B (Table 10), the final indirect effect is calculated as (-0.000210)*(-3.645064), which results in a value of 0.000765, according to the procedure from Moreira et al. (2016). This means that the indirect effect of the money per capita on optimal consumption, via relative prices, shows a directly proportional impact of money per capita on optimal consumption. Note that the values related to the indirect effects are recorded in Table 10 as "Indirect effect." The indirect effects for other models from Tables 9 and 10, 2A - 2B, 3A - 3B, and 4A – 4B, can be calculated with the same procedure.

Once more, based on Table 10, the empirical results show that money per capita variation impacts the optimal consumption variation, implying that the money is not neutral. We conclude that there are three empirical pieces of evidence that money is not neutral. The first shows that money influences relative prices (Table 9), and the second reveals that money also affects optimal consumption (Table 10). At last, there is an indirect effect of money (Table 9) affecting optimal consumption (Table 10), via relative prices (Tables 9 ad 10).

Conclusions

Based on the presented modeling, money becomes neutral only if changes in the money supply affect prices simultaneously and at the same proportion, as established by the QTM and implicitly by the standard consumer theory. In this case, money is neutral if the elasticities of money supply to the consumer price levels are equal.

On the other hand, money is not neutral if the elasticities of the money supply to the consumer price levels are unequal, regarding P_1 and P_2, for instance. Hence, Cantillon's approach is confirmed, and the money is not neutral. Besides, whether the money stock changes affect both prices in different intensities, the relative price (P1/ P2) also changes.

Summing up, once that change in the relative price resulting from an increase in the money stock alters the consumer's goods baskets, money is not neutral since it affects the relative prices in the economy, i.e., a real variable. It means that the dichotomy between the price level and the relative price is not valid. In this context, the standard consumer theory can be considered a special case from a more general consumer theory. In the general case, the elasticity of money supply to the consumer price levels are different, i.e., $\varepsilon_1 \neq \varepsilon_{2,}$ and consequently, money is not neutral. In the special case, the elasticities are equal, i.e., $\varepsilon_{1} = \varepsilon_{2,}$ and money is neutral.

In this context, according to several econometric models, considering quarterly data from 1946:04 to 2019:04, the empirical results show that the money stock variation directly affects the optimal consumption variation and indirectly affects the optimal consumption change via relative prices variation. Therefore, there is empirical evidence that money is not neutral since the elasticity of the money supply to the relative prices is not unitary. In other words, the elasticity of money supply to the consumer price levels are different, i.e., $\varepsilon_1 \neq \varepsilon_{2,}$ and consequently, money is not neutral in a non-conventional approach.

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